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The Two Dimensional XY Spin Glass with Ferromagnetic Next-Nearest-Neighbour Interactions

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ABSTRACT

The random-bond XY spin glass with ferromagnetic next-nearest-neighbour interactions is studied on a square lattice by Monte Carlo simulations. We find strong evidence for a finite-temperature spin glass transition at $T_c \approx 1.1$. We also give estimates for the spin glass critical exponents for different values of the strength of the nearest-neighbour interaction. Our results are consistent with universal behaviour.

Over the past two decades there has been considerable interest in spin glasses. There now exists substantial numerical evidence that the spin glass transition occurs at zero temperature in two dimensions (2d) for both Ising [1-5] and vector [6-9] spin glasses with nearest-neighbour interactions.

Spin glass order is widely believed to occur at a finite temperature for the Ising spin glass in three dimensions [3,4]. Hence, the lower critical dimension of the Ising spin glass is expected to be between 2 and 3 [10,11].

The two-dimensional Ising spin glass has been re-investigated in recent years [12,13]. Shirakura and Matsubara [12] have presented numerical evidence to suggest that the asymmetric random bond ($+J$ and $-0.8J$) Ising spin glass in 2d has a finite-temperature spin glass phase transition.

Further evidence for a non-zero transition for the Ising spin glass in 2d came recently from a Monte Carlo study by Lemke and Campbell [13] who modified the Edwards-Anderson model to include ferromagnetic interactions between next-nearest-neighbours. Lemke and Campbell [13] found that the modified 2d Edwards-Anderson Ising spin glass exhibits behaviour very similar to that seen in conventional Ising spin glasses in 3d [4].

For vector spin glasses, such as XY spin glasses, the situation is somewhat more controversial and complicated because of the presence of chirality. The numerical evidence [6-9] clearly points to a zero-temperature transition in the nearest-neighbour XY spin glass in 2d with both Gaussian and random ($\pm J$) bond distributions. The chiral-glass transition is also believed to occur at zero-temperature [8,14,15]. However, it has been argued that the chiral and phase variables decouple on long length scales. As a consequence, the values of the chiral- and spin-glass correlation-length exponents are significantly different [8,15].

Although most work would seem to point towards a zero-temperature phase transition also in three dimensions [6,7,16], some existing Monte Carlo data can be fitted equally well assuming a finite temperature transition [7].

Very recently, Maucourt and Grempel [17] have used a domain wall renormalisation

group method to suggest that the lower critical dimension of the XY spin glass model is very close to three, even possibly three itself. The results, however, point to a finite-temperature chiral-glass transition in 3d [17].

Given the recent developments for Ising spin glasses, in this Rapid Communication we re-visit the XY spin glass in 2d. Here we concentrate on the spin glass transition. We shall present numerical evidence that including next-nearest-neighbour ferromagnetic interactions in a random-bond XY spin glass in 2d induces a finite-temperature spin glass transition.

The Hamiltonian for the modified model [13] we study is given by

$$H = -\sum_{<i,j>} J_{ij}\mathbf{S}_i.\mathbf{S}_j \; - \sum_{<<k,m>>} \mathbf{S}_k.\mathbf{S}_m, \tag{1}$$

where $\mathbf{S}_i$ are planar spins on a square lattice of size L^2 ($L \leq 12$) with periodic boundary conditions. Whereas the first summation $< i,j >$ runs over all nearest-neighbour pairs only, the second summation $<< k,m >>$ denotes sums over all second nearest-neighbour pairs a distance $\sqrt{2}$ apart. The interactions J_{ij} are quenched independent random variables chosen from the following binary distribution:

$$P(J_{ij}) = \frac{1}{2}[\delta(J_{ij} - \lambda) + \delta(J_{ij} + \lambda)]. \tag{2}$$

In our simulations we consider $0 \leq \lambda \leq 1$. Clearly, for $\lambda = 0$ the lattice decouples into two independent inter-penetrating square sub-lattices. Each sub-lattice is a pure XY model which will undergo a Kosterlitz-Thouless [18] phase transition at $T \approx 0.89$ [19].

In our Monte Carlo simulations we study the dimensionless Binder parameter g_{SG} given by [4]

$$g_{SG} = 3 - 2\frac{q^{(4)}_{SG}}{(q^{(2)}_{SG})^2}, \tag{3}$$

where $q^{(2)}_{SG}$, the spin glass order parameter, and $q^{(4)}_{SG}$ are defined by

$$q^{(2)}_{SG} = \frac{1}{N^2}\sum_{i,j}[< \mathbf{S}_i.\mathbf{S}_j >^2_T]_J \tag{4}$$

and

$$q_{SG}^{(4)} = \frac{1}{N^4} \sum_{i,j,n,p} [< \mathbf{S}_i.\mathbf{S}_j\mathbf{S}_n.\mathbf{S}_p >_T^2]_J \tag{5}$$

and here $< ... >_T$ denotes a thermal average, $[...]_J$ indicates an average over disorder and $N = L \times L$.

The spin glass susceptibility, χ_{SG}, is related to $q_{SG}^{(2)}$ by

$$\chi_{SG} = N q_{SG}^{(2)}. \tag{6}$$

We analyse our results according to standard finite size scaling [4]. Near a transition temperature T_c the Binder parameter is expected to scale as

$$g_{SG}(L,T) = \overline{g}_{SG}(L^{1/\nu}(T - T_c)), \tag{7}$$

where $\overline{g}_{SG}$ and ν are the scaling function and the correlation length exponent, respectively. The scaling form of the Binder parameter can be used to determine the value of T_c as $g_{SG}(L, T_c) = \overline{g}_{SG}(0)$ is independent of the system size L.

The scaling form of χ_{SG} is given by

$$\chi_{SG}(L,T) = L^{2-\eta}\overline{\chi}_{SG}(L^{1/\nu}(T - T_c)), \tag{8}$$

where $\overline{\chi}_{SG}$ is the scaling function and η is the exponent describing the power-law decay of correlations at T_c. The value of η can be determined from

$$\chi_{SG}(L, T_c) = L^{2-\eta}\overline{\chi}_{SG}(0). \tag{9}$$

We now turn to our computer simulations and discuss the results. We use Metropolis dynamics and sequential updating in our Monte Carlo simulations. The method of Bhatt and Young [4] is used to ensure that thermal equilibrium is achieved in the simulations.

For each value of L ($4 \leq L \leq 12$) and λ ($0.0 \leq \lambda \leq 1.0$) we averaged over 500 pairs of samples. As most of the simulations were performed over a relatively high temperature

range ($0.7 \leq T \leq 1.4$), we did not encounter any serious equilibration problems. In fact, we were able to achieve equilibrium within 6400 Monte Carlo steps for the largest lattice at the lowest temperature considered. Nevertheless, the simulations presented in this work took approximately 3 months of CPU time distributed over 10 Silicon Graphics workstations.

Plots of the Binder parameter for various values of λ are shown in figures 1(a)-(e). The statistical error-bars are in most cases smaller than the size of the data points. Each figure shows the data against the temperature for the four different values of L ($L = 4, 6, 8$ and 12). The curves for $\lambda = 1.0$ (figure 1(a)), $\lambda = 0.7$ (figure 1(b)), $\lambda = 0.5$ (figure 1(c)) and $\lambda = 0.3$ (figure 1(d)) clearly intersect and splay out below the intersection point. This strongly suggests a spin glass transition. Both the transition temperature, $T_c(\lambda)$, and the value of the Binder parameter, $g_{SG}(T_c)$, would appear to depend only marginly, if at all, on λ. Although there is some uncertainty in the intersection points for all of the values of λ, our data are consistent with a universal (i.e. λ-independent) transition temperature. This is to be contrasted with the non-universal behaviour found by Lemke and Campbell [13] in the Ising version of the model. The values of $T_c(\lambda)$ are summarised in Table 1 which also contains our estimates for the critical exponents (see below).

The curves in figure 1(e), which shows the plot for $\lambda = 0.0$, coalesce at around $T \approx 0.9$ and then remain together for all lower temperatures. (For this particular value of λ we went down to a lower temperature, $T = 0.5$.) This is, of course, what one expects to occur at the Kosterlitz-Thouless [18] transition.

We next extract the critical exponents from the data. For illustrative purposes, we show the scaling plots for $\lambda = 0.5$ only. Assuming that $T_c(\lambda = 0.5) = 1.1$, in figure 2 we give a log-log plot of χ_{SG} against L. The slope of the straight line yields a rather large value of $\eta = 0.96 \pm 0.05$. The values we obtained for η for the other cases of λ can be found in Table 1. Although there appears to be a lot of scatter in the values for $\eta(\lambda)$, given the uncertainties in $T_c(\lambda)$, our estimates are not incompatible with a universal value.

By setting the critical temperature to $T_c(\lambda = 0.5) = 1.1$ and considering ν as an

adjustable parameter, we estimate the correlation-length exponent to be $\nu(\lambda = 0.5) = 0.75 \pm 0.1$. The error quoted here is just an estimate of the range of values for which the data scale well. In figure 3 we show a scaling plot of the Binder parameter against $(T - 1.1)L^{1/0.75}$. We note that this value of ν is very similar to that found recently [20] for the *four-dimensional* XY spin glass. Our estimates for the correlation-length exponent for the other values of λ are given in Table 1. Once again, our results point to a universal value of ν.

Finally, as a consistency check, in figure 4 we display for $\lambda = 0.5$ a scaling plot of $\chi_{SG}/L^{2-\eta}$ against $(T - T_c)L^{1/\nu}$ with $T_c = 1.1, \eta = 0.96$ and $\nu = 0.75$. The large error-bars on this plot are a consequence of both the statistical errors in χ_{SG} and the uncertainty in the value of η. Nevertheless, all of the data appear to scale reasonably well.

To conclude, we have presented data from Monte Carlo simulations of a short-range XY spin glass in 2d with ferromagnetic next-nearest-neighbour interactions. We find strong evidence for a finite-temperature transition. We have used finite-size scaling to estimate the spin glass transition temperature and the critical exponents as functions of the strength of the nearest-neighbour interaction. Our results are consistent with universal behaviour. Furthermore, the value of the critical temperature and the correlation-length exponent are very similar to those found recently for the four-dimensional XY spin glass.

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Table 1

Estimates of the critical temperature, T_c, and the critical exponents, η and ν, for various different values of λ.

λ	T_c	η	ν
0.3	1.10 ± 0.05	0.85 ± 0.06	0.85 ± 0.05
0.5	1.10 ± 0.05	0.96 ± 0.05	0.75 ± 0.1
0.7	1.05 ± 0.10	1.08 ± 0.06	0.75 ± 0.05
1.0	0.90 ± 0.15	0.83 ± 0.05	0.90 ± 0.1

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IGURE CAPTIONS

Figure 1(a)

A plot of the Binder parameter, g_{SG}, against the temperature for $L = 4, 6, 8$ and 12 with $\lambda = 1.0$. The lines are just guides to the eye.

Figure 1(b)

The Binder parameter against the temperature for $\lambda = 0.7$. The lines are just guides to the eye.

Figure 1(c)

The Binder parameter against the temperature for $\lambda = 0.5$. The lines are just guides to the eye.

Figure 1(d)

The Binder parameter against the temperature for $\lambda = 0.3$. The lines are just guides to the eye.

Figure 1(e)

A plot of g_{SG} against the temperature. The lines, which are just to guide the eye, clearly meet and then remain together for lower temperatures.

Figure 2

A log-log plot of the spin glass susceptibility against the linear dimension of the lattice for $\lambda = 0.5$. The slope of the straight line gives us a value of $\eta = 0.96 \pm 0.05$.

Figure 3

A scaling plot of g_{SG} against $(T - T_c)L^{1/\nu}$ with $T_c = 1.1$ and $\nu = 0.75$ (here $\lambda = 0.5$).

Figure 4

A scaling plot of $\chi_{SG}/L^{2-\eta}$ versus $(T-T_c)L^{1/\nu}$ for $\lambda = 0.5$ assuming that $T_c = 1.1, \nu = 0.75$ and $\eta = 0.96$. The solid line is just to guide the eye.

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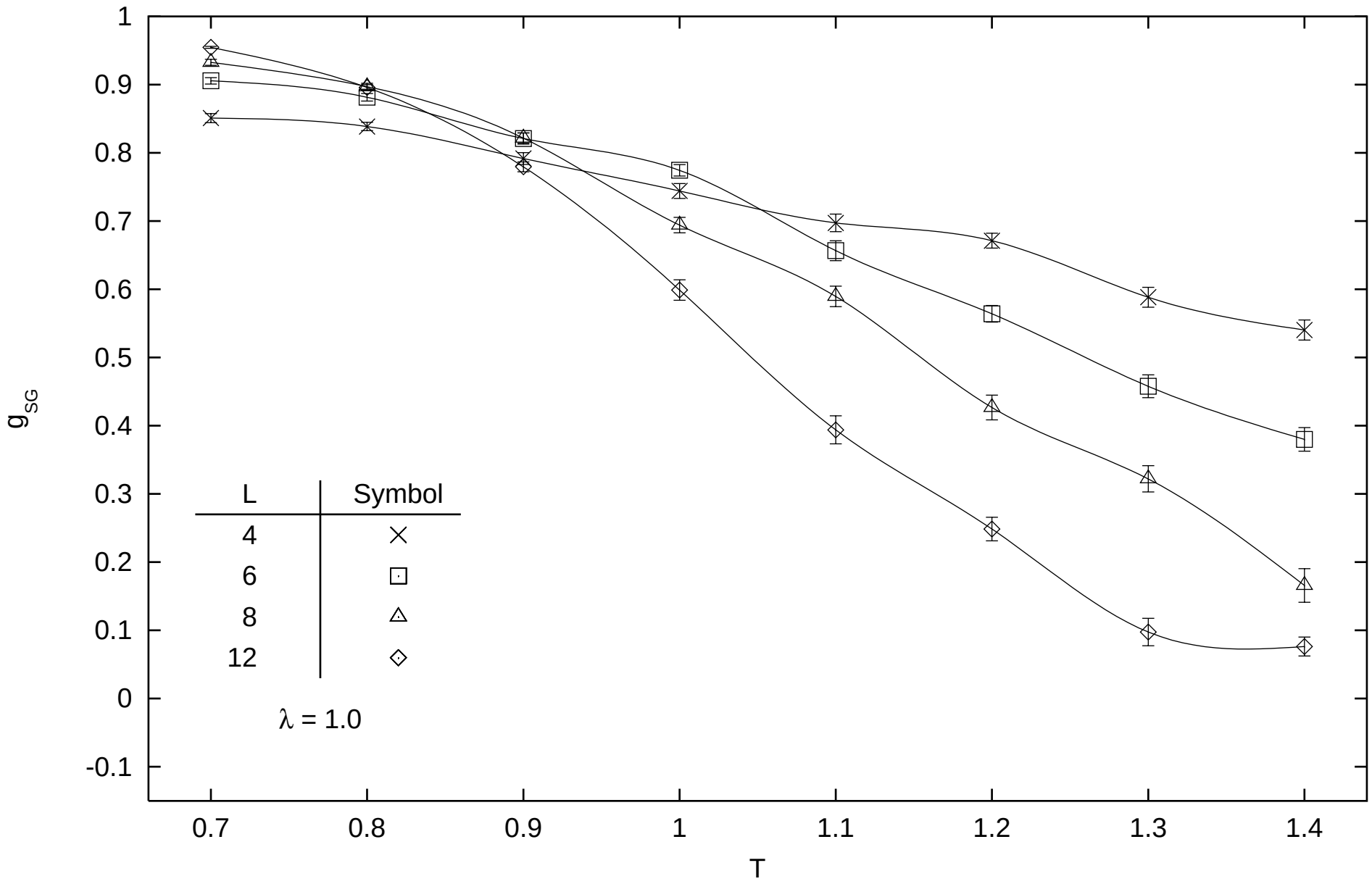
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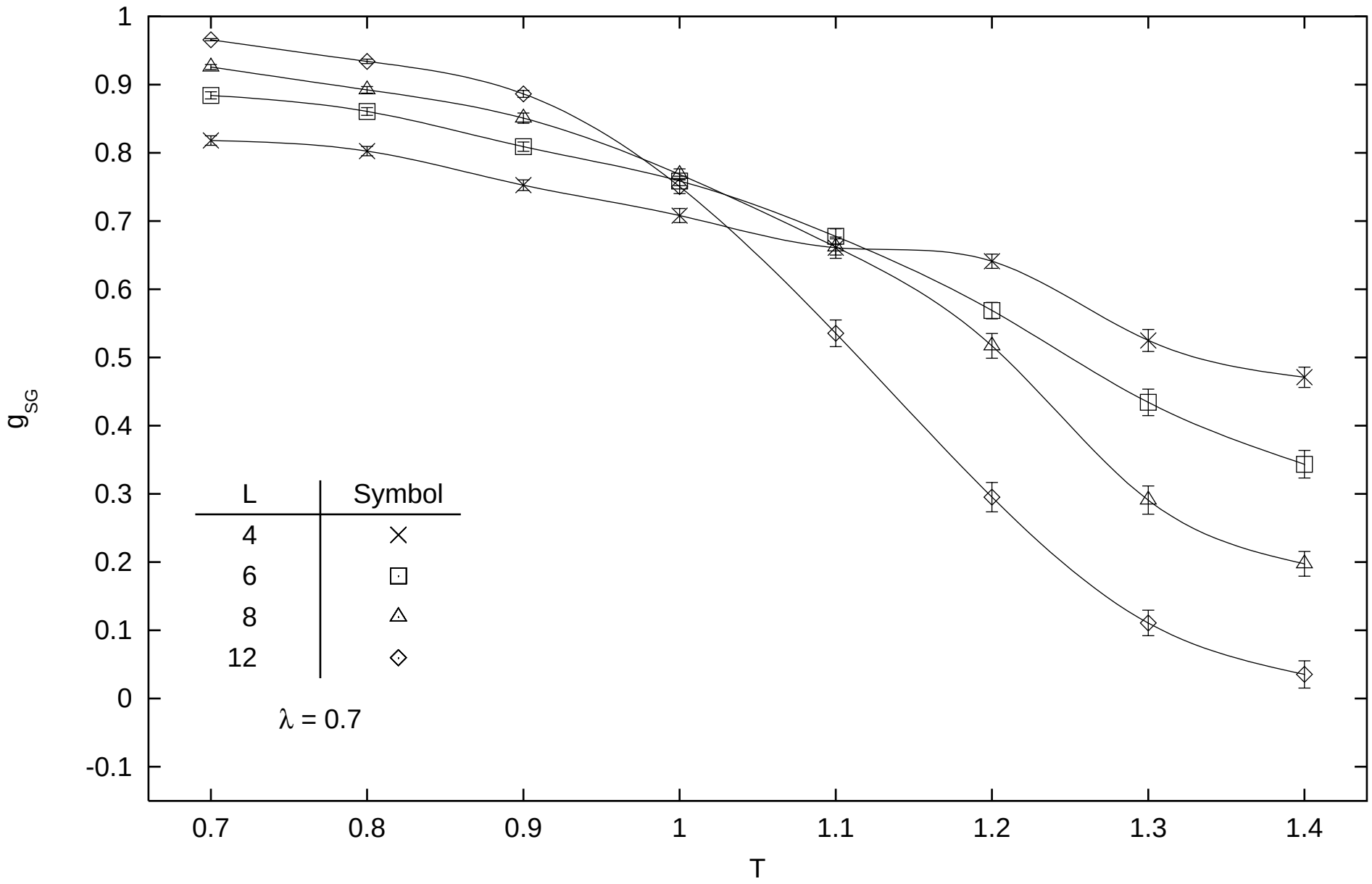
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
g_{SG}
0.7
0.8
0.9
1
1.1
1.2
1.3
1.4
T
L
Symbol
4
6
8
12
λ = 1.0

Figure 1(a)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-0.1
g_{SG}
0.7
0.8
0.9
1
1.1
1.2
1.3
1.4
T
L
Symbol
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6
8
12
λ = 0.7

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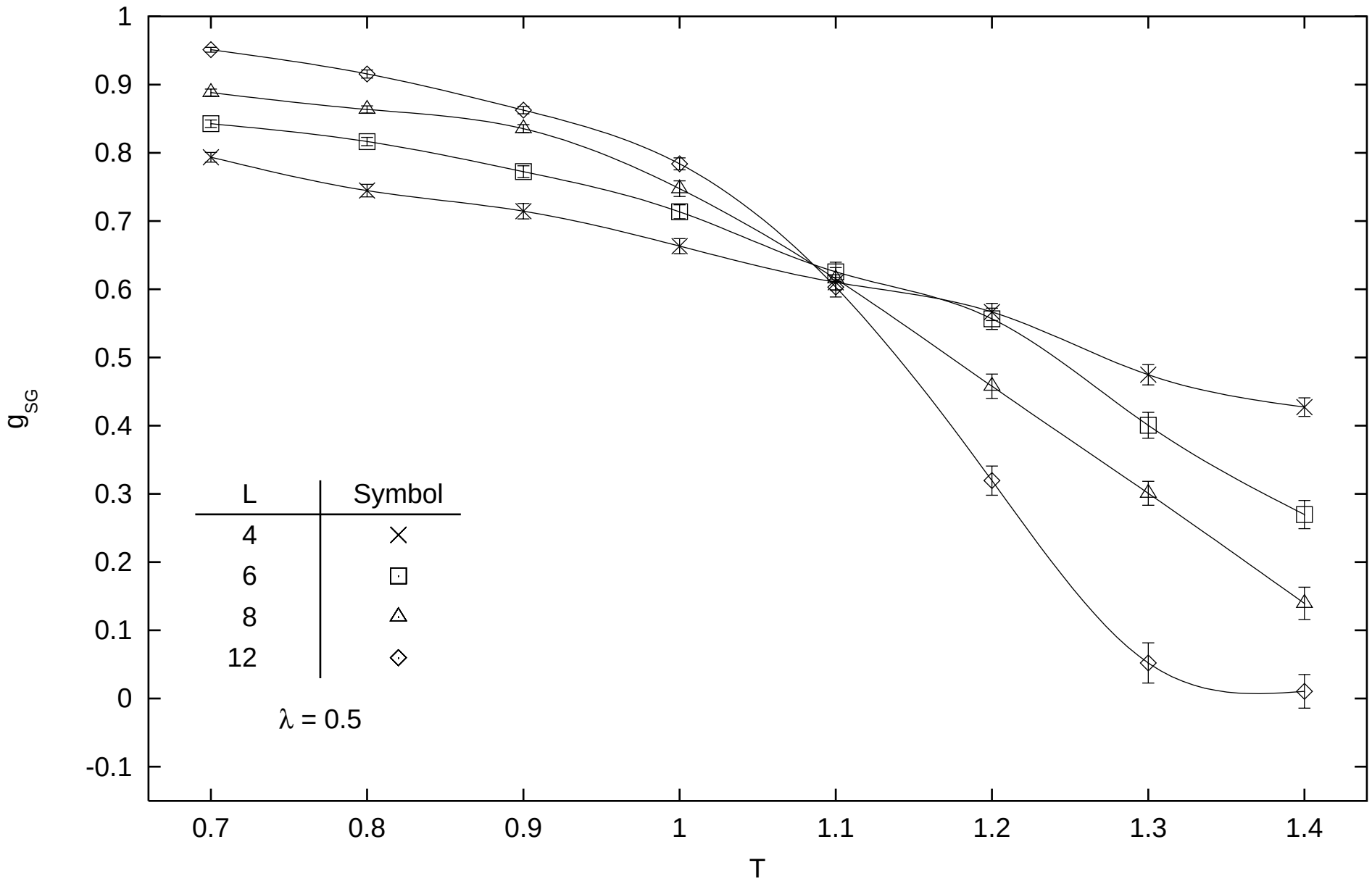
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0.2
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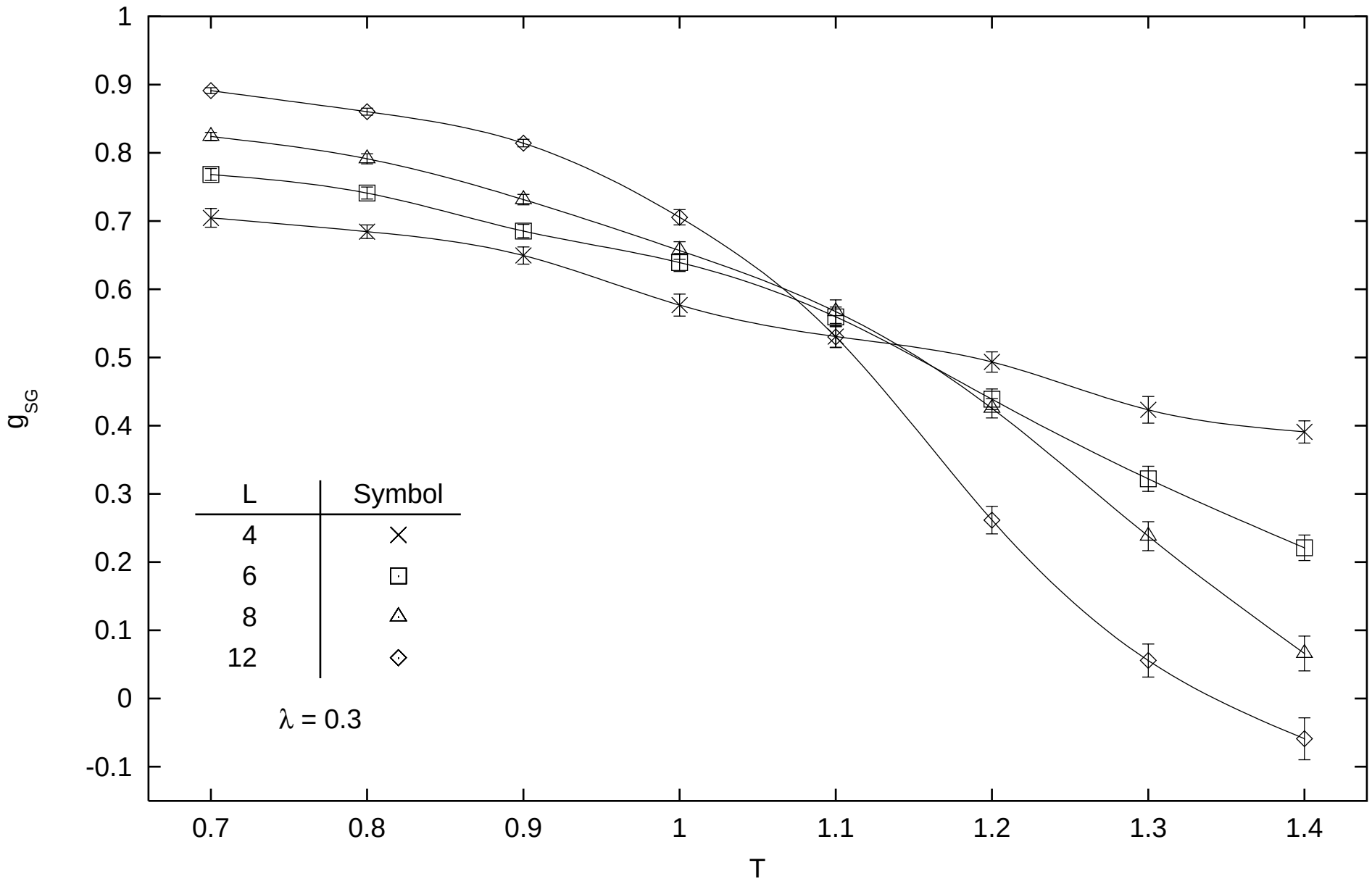
0
-0.1
g_{SG}
0.7
0.8
0.9
1
1.1
1.2
1.3
1.4
T
L
Symbol
4
6
8
12
λ = 0.5

Figure 1(c)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0.7
0.6
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0
-0.1
g_{SG}
0.7
0.8
0.9
1
1.1
1.2
1.3
1.4
T
L
Symbol
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6
8
12
λ = 0.3

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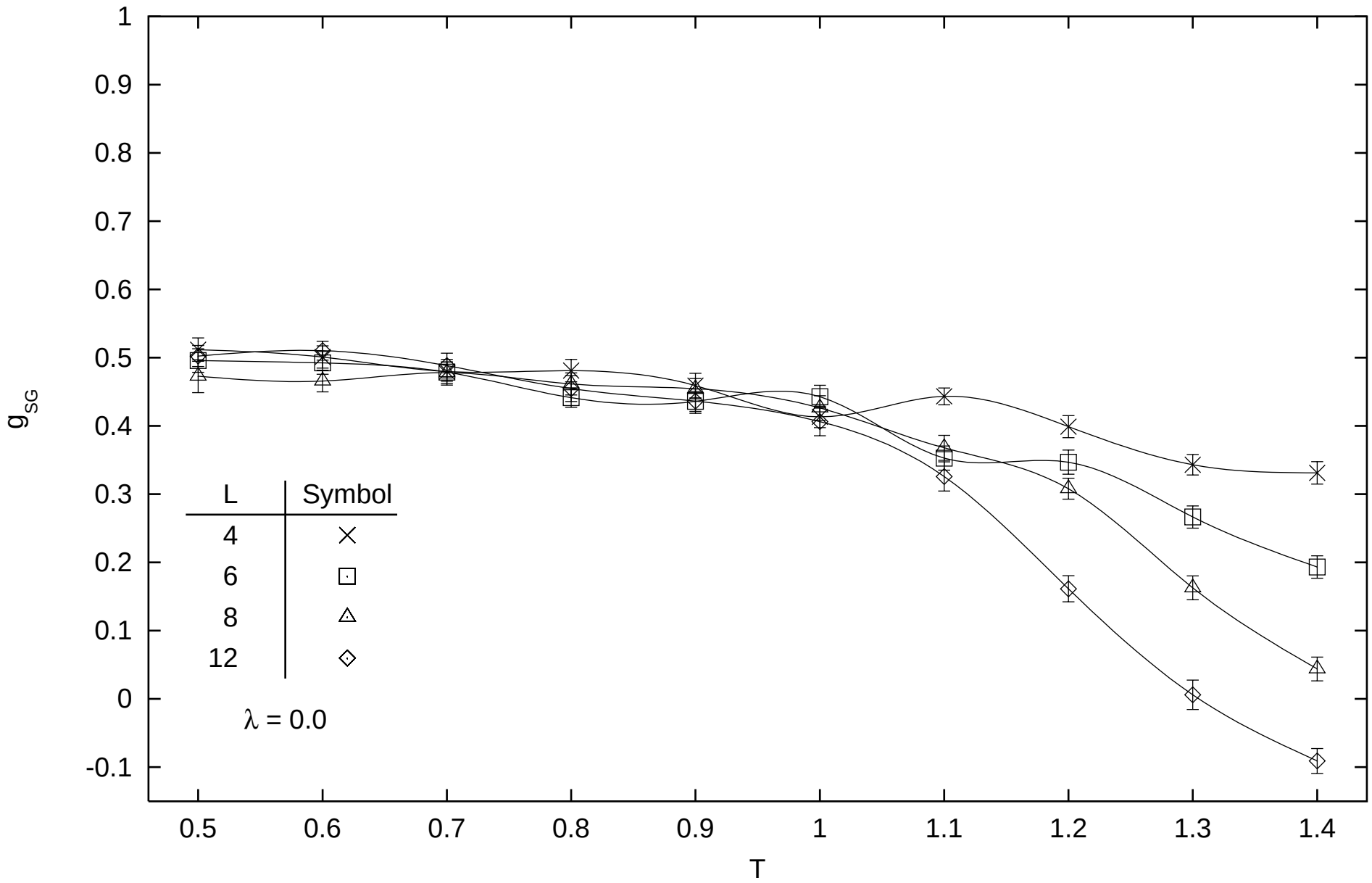
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0.7
0.6
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0.4
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0
-0.1
g_{SG}
0.5
0.6
0.7
0.8
0.9
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1.2
1.3
1.4
T
L
Symbol
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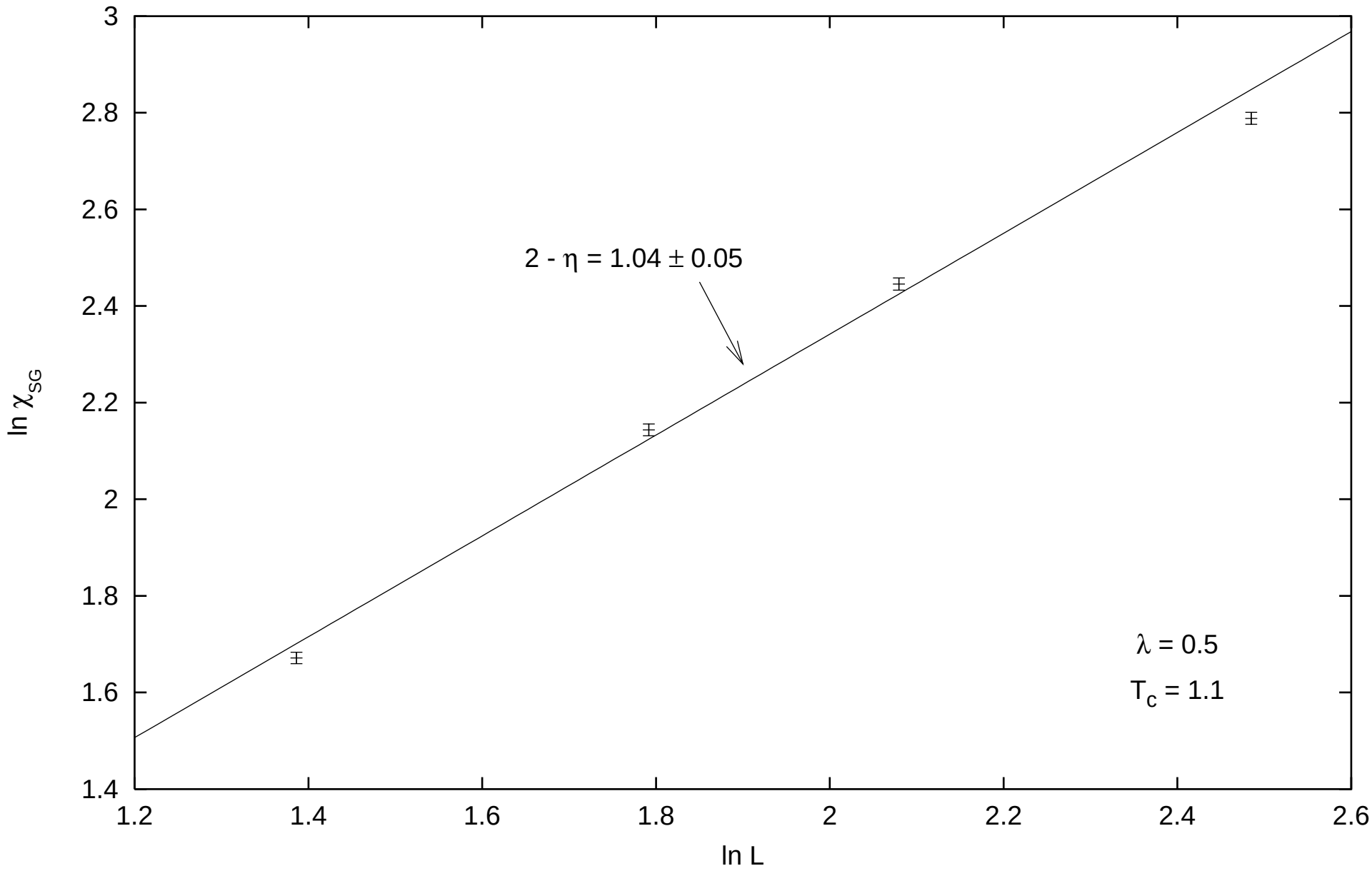
6
8
12
λ = 0.0

Figure 1(e)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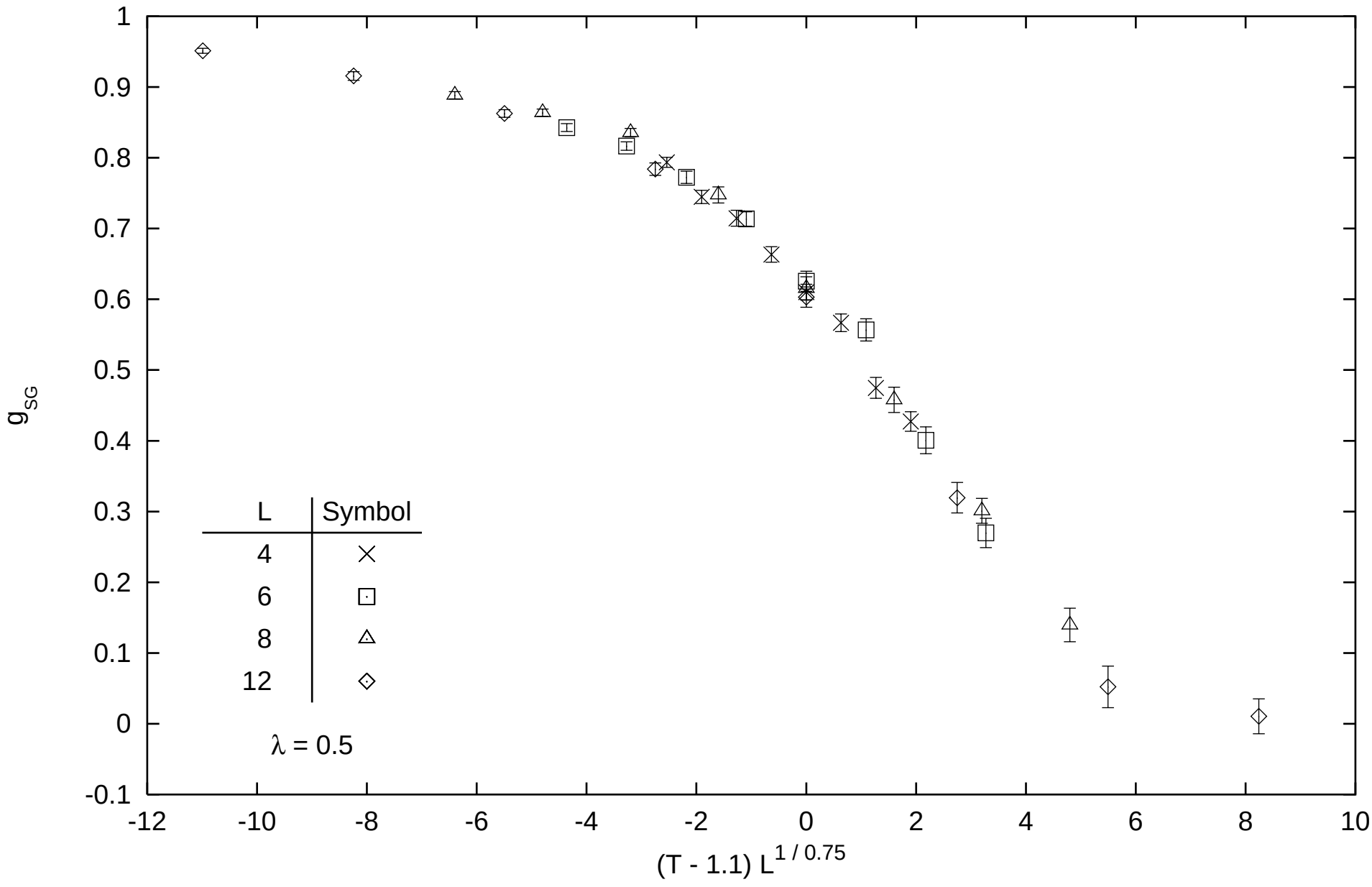
2.8
2.6
2.4
2.2
2
1.8
1.6
1.4
1.2
1.4
1.6
1.8
2
2.2
2.4
2.6
ln χ_{SG}
ln L
2 - η = 1.04 ± 0.05
λ = 0.5
T_c = 1.1

Figure 2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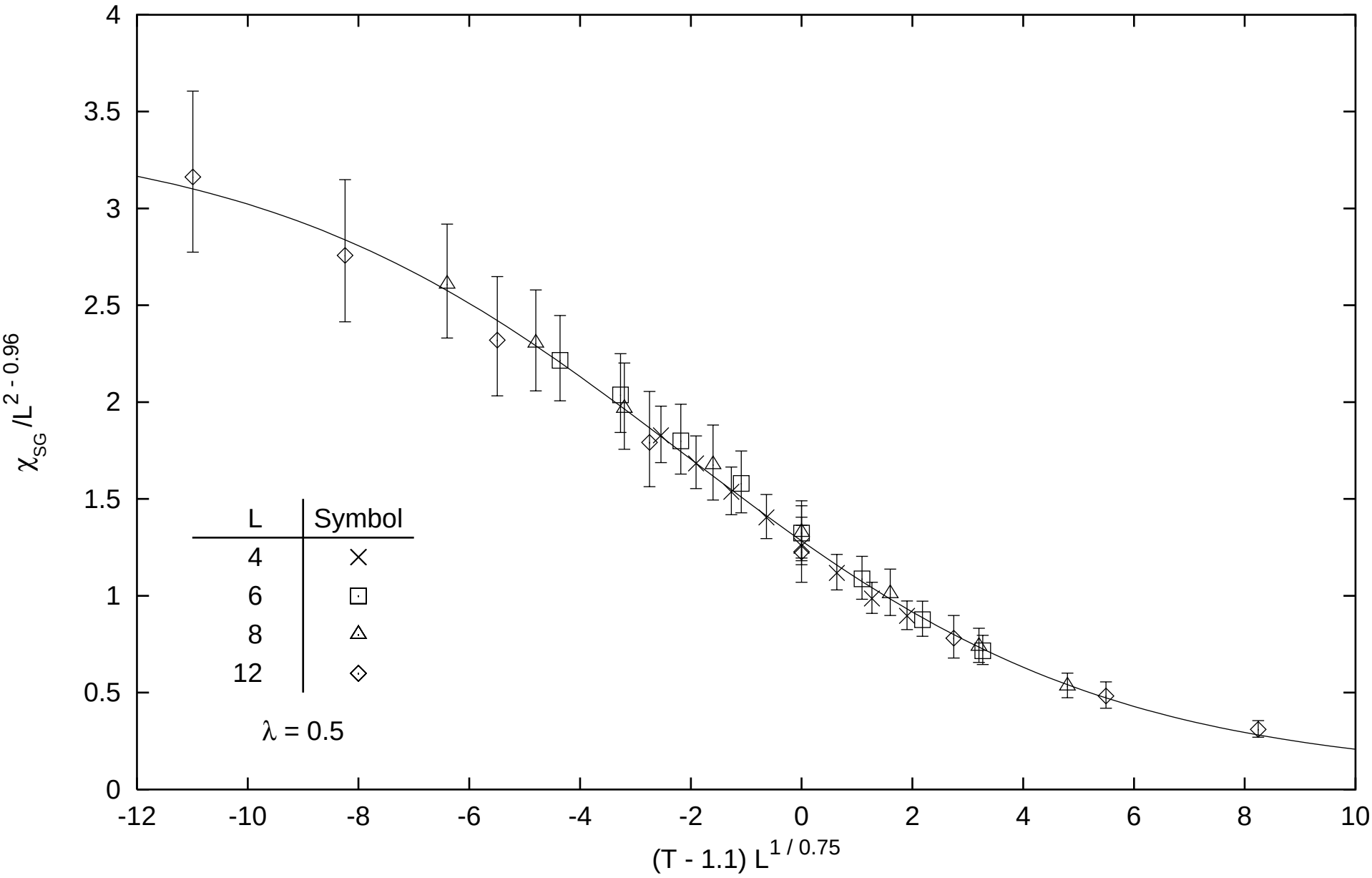
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
g_{SG}
$(T - 1.1)\ L^{1 / 0.75}$
L
Symbol
4
6
8
12
λ = 0.5

Figure 3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3
2.5
2
1.5
1
0.5
0
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
$\chi_{SG}/L^{2 - 0.96}$
$(T - 1.1)\ L^{1 / 0.75}$
L
Symbol
4
6
8
12
λ = 0.5

Figure 4